

### 3.10 Stochastic volatility models for financial time series

**Model description** Stochastic volatility models are used in mathematical finance to describe the evolution of asset returns, which typically exhibit changing variances over time. As an illustration we use a time series of daily pound/dollar exchange rates  $\{z_t\}$  from the period 01/10/81 to 28/6/85, previously analyzed by Harvey, Ruiz & Shephard (1994). The series of interest are the daily mean-corrected returns  $\{y_t\}$ , given by the transformation

$$y_t = \log z_t - \log z_{t-1} - n^{-1} \sum_{i=1}^n (\log z_t - \log z_{t-1}).$$

The stochastic volatility model allows the variance of  $y_t$  to vary smoothly with time. This is achieved by assuming that  $y_t \sim N(\mu, \sigma_t^2)$ , where  $\sigma_t^2 = \exp(\mu_x + x_t)$ . The smoothly varying component  $x_t$  follows the autoregression

$$x_t = \beta x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2).$$

The vector of hyper-parameters is for this model is thus  $(\beta, \sigma, \mu, \mu_x)$ .